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# DETERMINATION OF THE YOUNG'S MODULUS OF TOGO ROCKS BY THE MORI-TANAKA HOMOGENIZATION

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## ABSTRACT

The article presents the study of the application of the porous media model (case of the Mori-Tanaka homogenization model) in order to determine the Young's modulus of the granites, quartzites and gneisses taken from the quarries and outcrops of Togo. 91 rock samples including 49 granite and quartzite samples and 42 gneiss samples were studied. The complexity of the mechanical behavior of rocks, related to rock diversity from the petrographic, textural, structural and physical point of view, is demonstrated by weak correlations between these features. In this context, we propose the use of the classical and modified Mori-Tanaka models applied to the three groups of rocks (the set of samples, the granites and the quartzites, the gneisses). In the classical Mori-Tanaka method (E (YMT)), according to which the rock is considered to be a biphase solid medium (matrix + inclusion), the role of the matrix is attributed to feldspar, the majority mineral of the rocks, and the collection of other minerals represents inclusions. The second method proposes to consider the rock as a tetraphase medium (feldspar, quartz, biotite and amphibole) and the Young's modulus of the rock (E (YT)) is determined as the sum of the elemental values of the module of each mineral according to the classical method of homogenization of Mori-Tanaka.

For rocks with a Young's modulus of less than 70 Gpa – low resistance rocks – the application of the modified Mori-Tanaka method is of interest in the determination of the Young's modulus of gneiss, whereas of the classical Mori-Tanaka method gives satisfactory results for granites and quartzites. Neither method can be used to estimate the Young's modulus of granites, quartzites and gneisses, which Young's modulus is high (> 70 GPa).

**KEYWORDS:** Young's modulus, Mori-Tanaka model, granites, quartzites, gneisses.

## 1. INTRODUCTION

This article presents the study of the properties and mechanical behavior of rock materials and their response to a force applied in their physical environment. The construction work modifies the initial state of the constraints of the rock mass in a very short time compared to the geological formation time of the massif.

In road engineering, the choice of the pavement structure is based on two main criteria: the circulation and the characteristics of the foundation soil, such as its resistance to punching, its resistance to compression and its deformation (the modulus of elasticity).

The aim of our study is to evaluate the characteristics of the gneisses, granites and quartzites taken from the quarries and outcrops of these rocks in the territory of Togo and their influences on the Young's modulus, on the one hand, and to estimate the modulus of elasticity from the tests of compression mono axial, on the other hand, necessary for determination of the structure of the pavement in road construction. We seek from the knowledge of microstructure of the rock, which is heterogeneous, to determine the macroscopic, mechanical characteristics. The micro-macro particle approach (discrete element method) recommends the determination of the mechanical properties from the microstructure of the rock (Cundall, 1971 [1]), but one of the difficulties of these models resides in the calibration of the properties to the particle scale to find the macros properties. The macroscopic behavior of the model is governed by the density of the particle arrangement, particle size distribution, and properties of particles and contacts (Potyondy and Cundall, 2004 [2]). In this type of model, the numerical

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particle is associated with a rock sample and thus has homogenized mechanical properties that make it possible to reproduce the macroscopic behavior of the rock (Potyondy, Cundall, 2004 [2]). The studies and applications carried out by Mr. Peter-Borie and S. Gentier (2014 [3]) demonstrate the need to integrate the structural characteristics of rocks. The mineralogical composition of the rock is directly reflected in the number of families of particles and their characteristics: each grain type (quartz, feldspar, etc.) is associated with a family of particles whose proportions and granulometry are identical to those observed in the rock. Another approach currently used: the micromechanical modeling of the properties of composite materials, considered as heterogeneous and porous media, has been developed in the work of H. Lebail (2001 [4]), JL Auriautl (1991 [5]), J Sarout (2006 [6]) and L. Dormieux, D. Kondo, F. J. Ulm (2006[7]). The models (effective elastic media) based on the multifactorial dependences between the compression coefficient K, the shear coefficient μ, the mineralogical composition, the shape, the arrangement of the mineral grains and the Young's modulus were proposed by Kuster and Toksoz (Effective model, 1974 [8, 9]), Berryman (Differential effective media DEM, 1979 [10]) and O'Connell and Budiansky (Self-consistent approximation SCA, 1974, 1976 [11, 12]). This last model presents the rock like a matrix, in which the grains of the inclusions are randomly oriented having the shape of "dry penny-shaped cracks" was generalized for a medium containing N-phases (N-constituents) by Berryman (2005). [13]). Rocks, being heterogeneous and porous materials, we proceed, in this study, to homogenize the rocks by the Mori-Tanaka method which makes it possible to link the mechanical properties to the microstructure of the heterogeneous and porous media (A.Rahmouni, A. Boulanouar, 2009,2013 [14,15]) and to model the mechanical behavior of rocks (granites, quartzites and gneisses), taking into account their mineralogical, structural and physical characteristics.

## 2. MATERIALS AND METHODS

The samples of the rocks studied come from the outcrops and quarries of Togo, in total 91 samples. The rocks are of different types: quartzites, biotite and amphibole granites, quartzo-feldspathic gneisses, and biotite and amphibole gneisses. The main mineral of the rocks is feldspar whose content varies from 49.4 to 89.5%. The mechanical properties of rocks and aggregates, produced from the rocks, are determined by the Los Angeles and Micro-Deval tests in the presence of water and the uniaxial compressive strength was determined on 4x4x8 prismatic specimens. From the compressive strength and the value of the strain measured during the compression test, the value of the Young's modulus was determined by Hooke’s formula:

$$\boldsymbol{\sigma = E\varepsilon}$$

The results of laboratory tests are treated by statistical methods which prove to be fruitful and sometimes even indispensable. Among these methods, a primordial place is given to the statistical study of the reciprocal connections between variables. The mathematical tools used [16] in this area are essentially: correlation analysis and regression analysis. Regression analysis makes it possible to deduce an estimation equation that describes the functional nature of the relationship between two variables, while the correlation analysis gives a measure of the strength of this relationship.

In this study, statistical analysis is used to determine the correlations between petrographic, physical and mechanical characteristics of granites, gneisses and quartzites.

The Mori-Tanaka homogenization method associated with the rock characteristics will allow the determination of Young's modulus of granites, gneisses and quartzites from Togo.

According to the Mori-Tanaka method, an elastic, linear and isotropic bi-phasic medium consists of a matrix containing inclusions. The elasticity tensor, the compression and shear moduli of the phase i (i = matrix, inclusion) are respectively Ci, ki, and μi. The macroscopic behavior is then elastic, linear and isotropic. Tensors of stress and macroscopic strain Σ and E are connected by the equation:

$$\Sigma = C^{hom} : E \qquad (1)$$

In the case of a heterogeneous medium consisting of a material containing inclusions, the elastic tensor estimate is written [14]:

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CsecMT=Cm +finc fmSE :C-1+ (Cinc -Cm)-1 $^{-1}$ (2)

$C_m$ –the elastic tensor of the matrix,
$C_{inc}$ – the elastic tensor of inclusions,
$S^E$ – Eshelby tensor,
$f_m$ – the volume fraction of matrix,
$f_{inc}$ – the volume fraction of inclusions.
In the case of spherical inclusions, we have :
ksecMT= km 1+finckinc-kmkm+fmkinc-km (3)

secMT= m 1+fincinc-mm+ fminc-m(4)

$k_m$ –compression coefficient of the matrix,
$k_{inc}$ – compression coefficient of inclusion,
$\mu_m$ – shear coefficient of the matrix,
$\mu_{inc}$ –shear coefficient of inclusion.

With :
=3km3km +4m  =6(km+2m)53km+4m (5)

By setting kinc = 0 and μinc = 0, we obtain the compression and macroscopic shear modules for a dry porous medium:
ksecMT= km (1- fp 3km+ 4m3fpkm+4m) (6)
sec MT=m 1-fp15km + 20m9+6fp km+8+12fp m (7)
With $f_p$ is porosity.

Young's module is written:
EsecMT= E01-fp/1+afp (8)
E0=9kmm3km+m  a=3km(33km+56m)(12km+4m)(9km +8m) (9)

km=E31-2v et m=E2(1+v) (10)
With E - Young's modulus of the mineral,
v - Poisson's ratio of the mineral.

Table 1 shows the determined values of the mineral parameters.

*Table 1. Mineral parameter values*

| Minerals | E , Gpa | v | $k_m$ | $\mu_m$ |
|---|---|---|---|---|
| Feldspar | 77 | 0.29 | 61.11 | 29.84 |
| Quartz | 73 | 0.17 | 36.86 | 31.19 |
| Biotite | 67 | 0.3 | 55.83 | 25.76 |
| Amphibole | 110 | 0.29 | 87.30 | 42.63 |

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## 3. RESULTS AND DISCUSSIONS

The petrographic characteristics, density, porosity of all rock types sampled (quartzites, gneisses and granites) [17]) and the mechanical properties [17, 18] allowed the study of correlations between intrinsic and mechanical properties using the MATLAB software "corrcoef" program [16]. The results obtained show that:

- The mechanical characteristics LA, MDE and the rock strength correlate with the quantity of quartz,
- The Young's modulus correlates with the quantities of Quartz, LA, MDE and rock strength (Pm, the depth of perforation determined by Rock Drilling Test).

Figures 1, 2, 3, 4 show point clouds and trend curves for all rocks sampled between mechanical characteristics (LA, MDE and rock strength) and quartz content, Young's modulus, and mechanical properties, the mineralogical composition and the physical properties of the rocks.

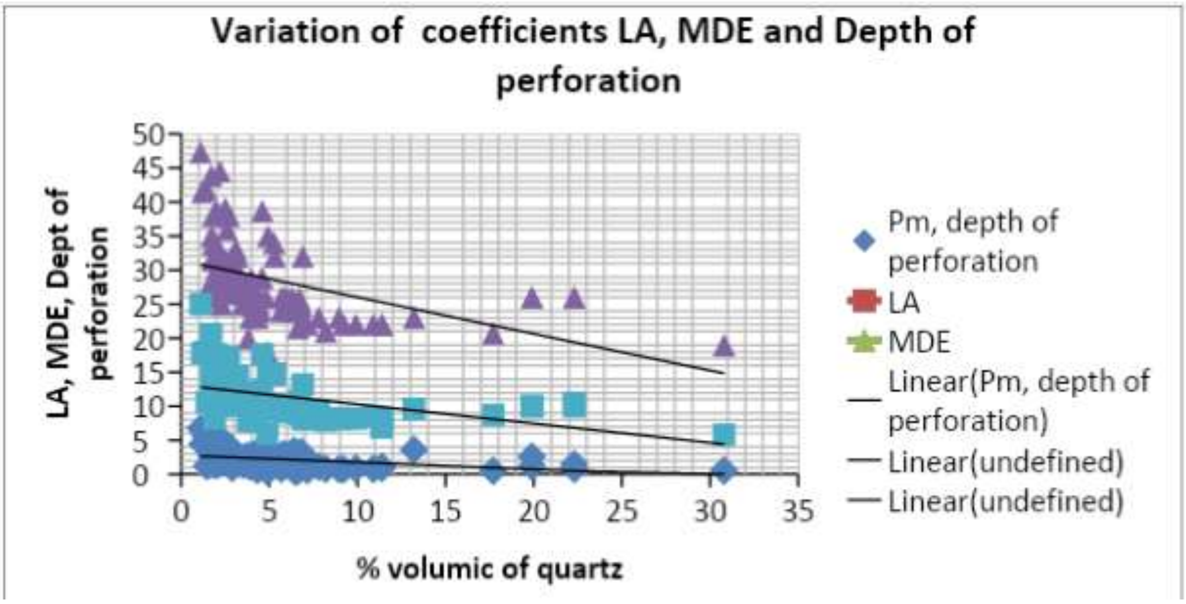


*Figure 1. Variation of coefficients LA, MDE and Rock Strength according to the quantity of Quartz*

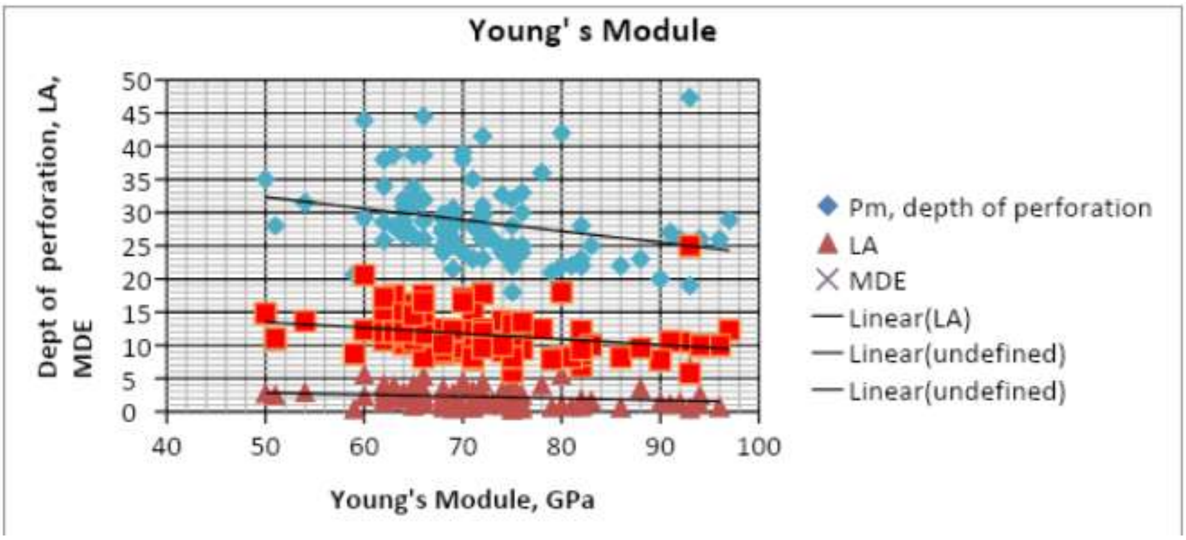


*Figure 2. Young's modulus variation according to the mechanical properties of the rock*

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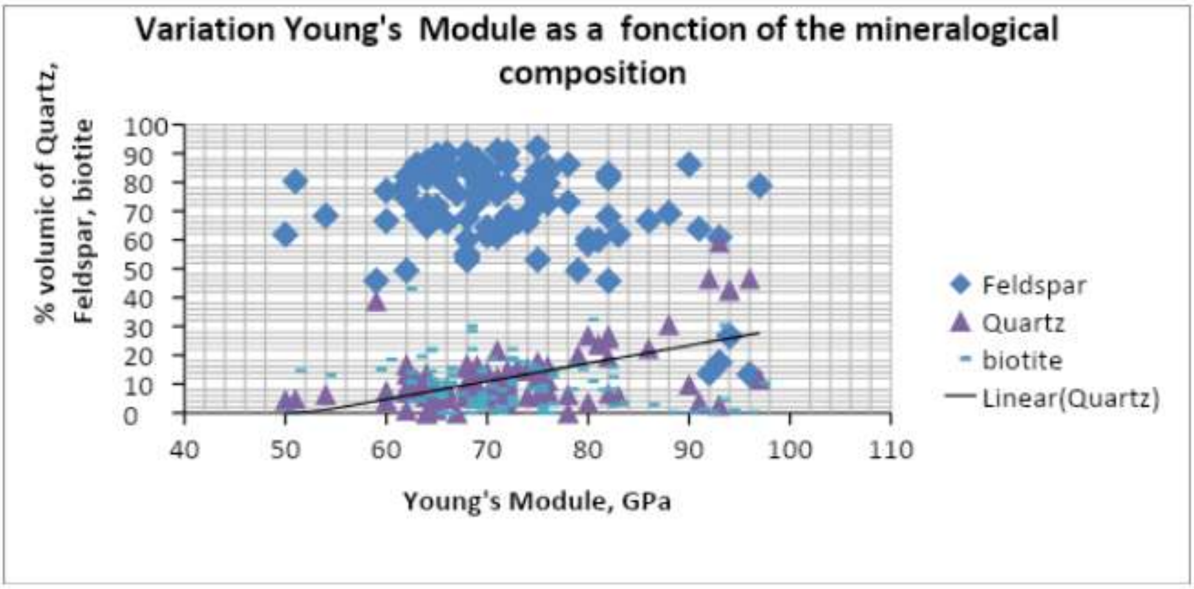


*Figure 3. Young's modulus variation as a function of the mineralogical composition of the rock*

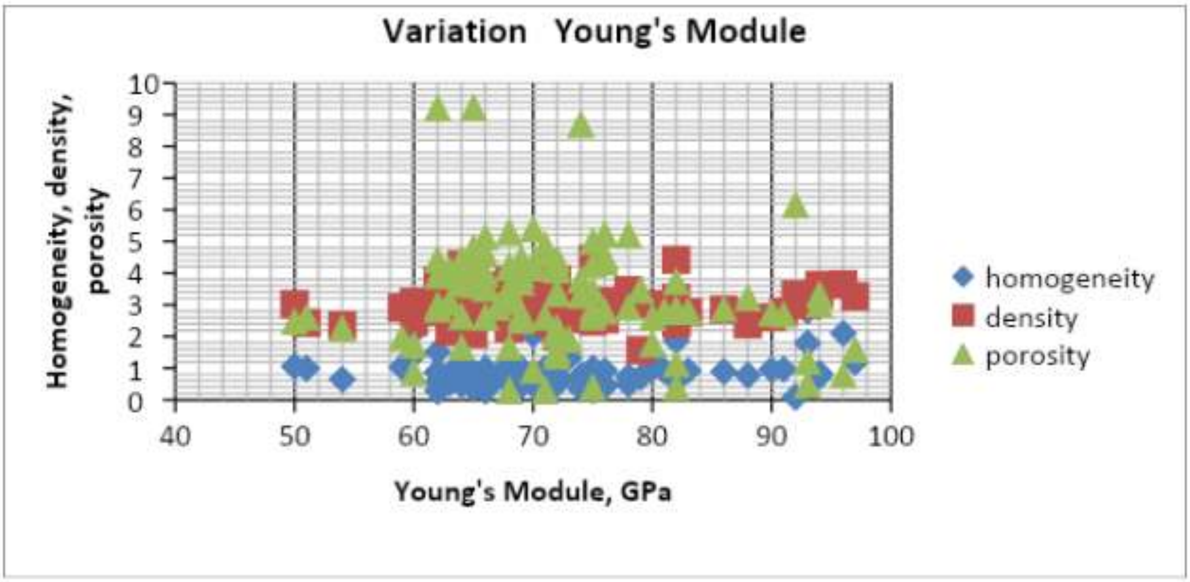


*Figure 4: Young's modulus variation according to the physical properties of the rocks*

The results of the tests and the characteristics of the rocks represented in the above figures show that the Young's modulus depends on the Quartz content, the LA and MDE coefficients: the increase in the quantity of Quartz and the decrease in LA coefficients and MDE and rock strength, increases Young's modulus. Nevertheless, Young's modulus scatter plot in terms of physical, mineralogical and mechanical properties of granites, gneisses and quartzites are dispersed globally.

No characteristic taken separately can therefore be used for the determination of the Young's modulus. The Young's modulus, , does not depend only on a characteristic, but on the set of parameters of the rock. This leads us to adopt the Mori-Tanaka homogenization system taking into account the texture, structure and mineralogical composition of granites, gneisses and quartzites. Since granites and quartzites have the equant texture and gneiss the foliated texture, three groups of samples were examined:

1. The set of rocks,
2. Granites and quartzites,
3. The gneisses.

To each group the Mori-Tanaka homogenization method was applied by considering the Feldspar mineral as the element of the matrix and the other minerals as inclusions and the Young Mori-Tanaka (EYMT) modulus was determined. Then, it has been proposed to determine the Young's modulus as the sum of the elementary Young Modules of all the minerals constituting the rock, considering each mineral as the matrix and the others as excluding (EYT) according to the formula:

EYT=i=1nEi (11)

With,

$E_{YT}$ : Young's modulus of the rock,

n: number of minerals in the rock,

Ei: Young's modulus of a mineral, considered mineral matrix.

The following Figures 5, 6, 7 represent the Young modulus values measured and determined according to the two methods for each group of rocks in accordance with the volume percentage of Feldspar.

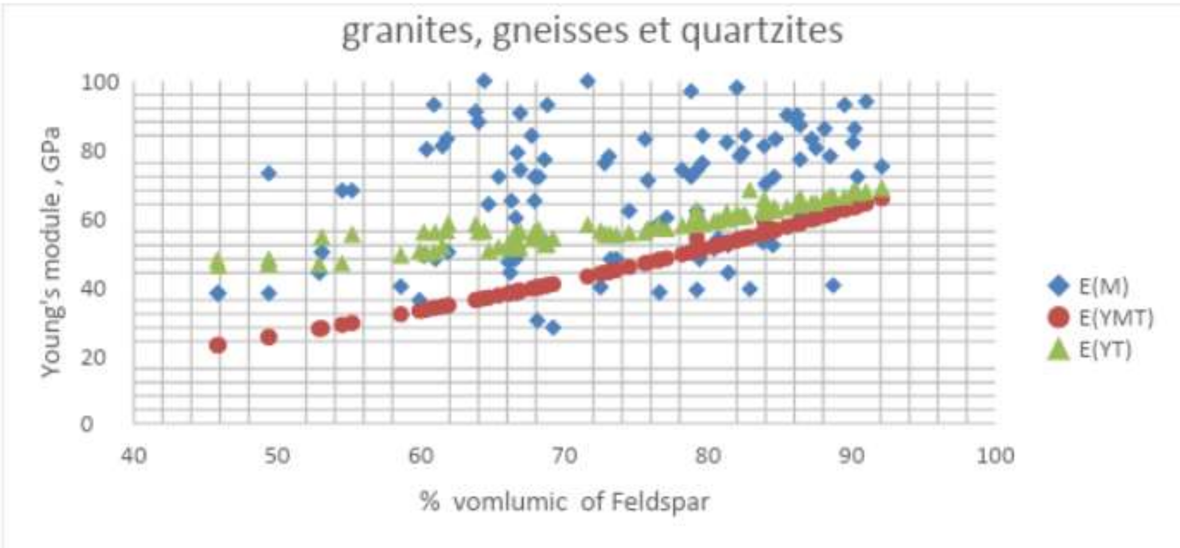


***Figure 5. Young's modulus variation as a function of % volumic Feldspar for all samples***

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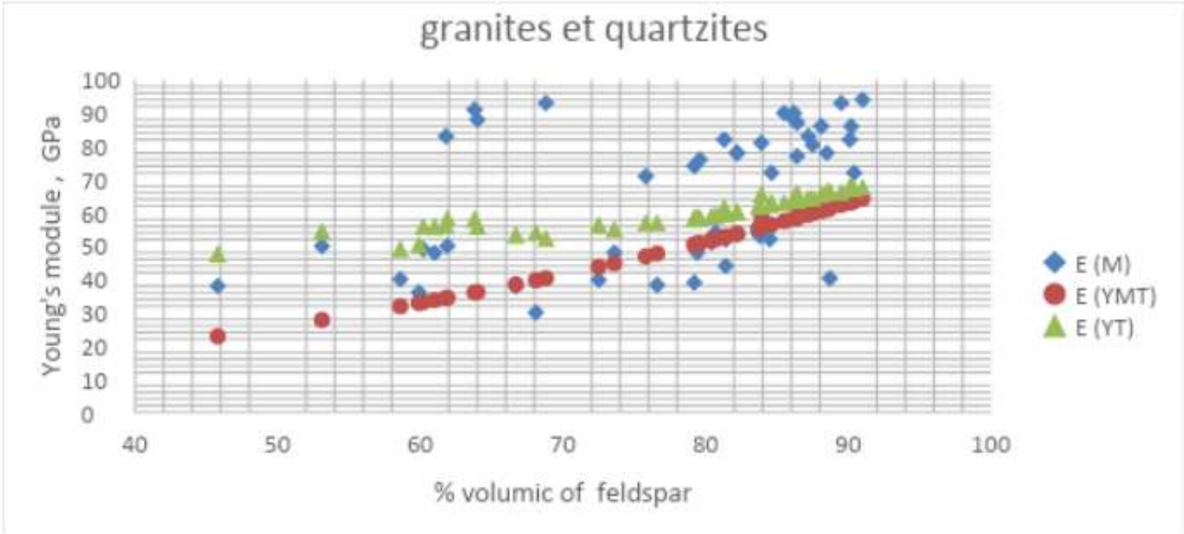


***Figure 6: Young's modulus variation as a function of % volumic Feldspar for granite and quartzite samples***

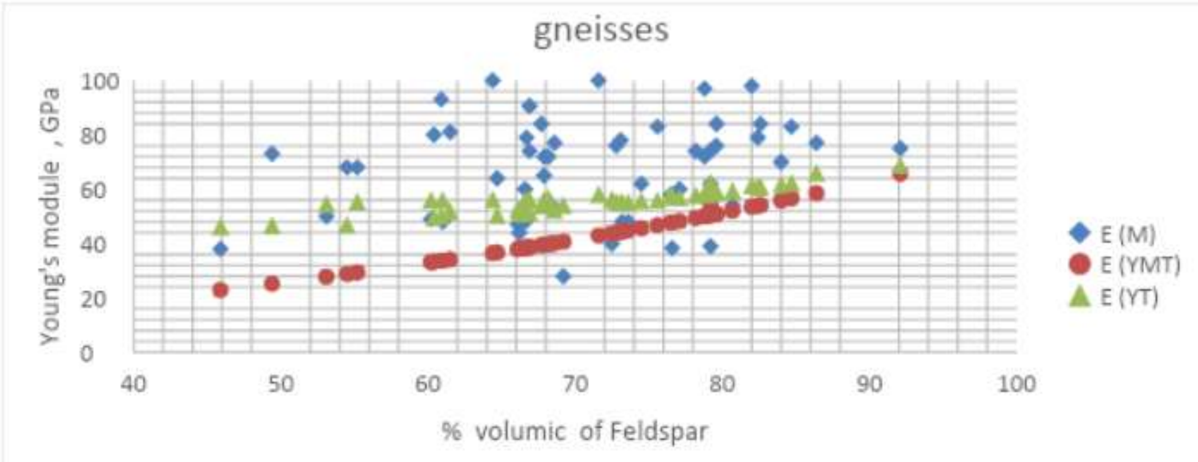


***Figure 7. Young's modulus variation as a function of % volumic Feldspar for gneiss samples***

The Young modulus values determined by the two methods increase as the percentage of feldspar increases: for small values of feldspar quantity the difference is of the order of 20 GPa; for high values (> 70% Feldspar) the difference is of the order of 5 GPa and decreases with the increase in the amount of Feldspar. The measured Young's modulus scatter plot is found above the values estimated by the Mori-Tanaka homogenization method and for each group of rocks.

Figures 8, 9, 10 show the Young modulus values measured and determined by the two methods for each group of rocks and allow to appreciate the differences between these values. The samples are ranked according to the measured Young's modulus increase order.

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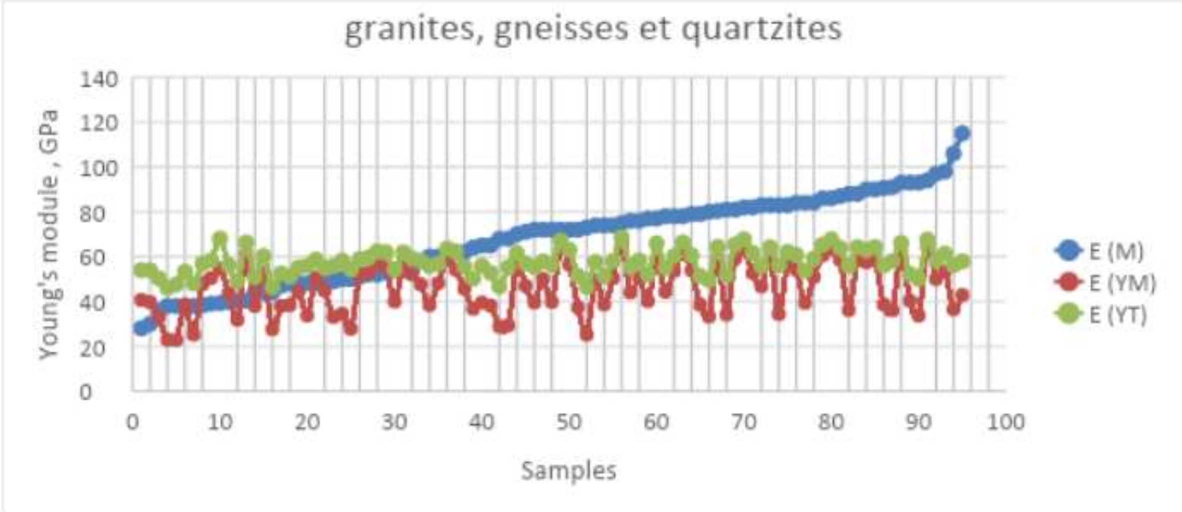


*Figure 8. Young's modulus variation of samples for all types of rocks*

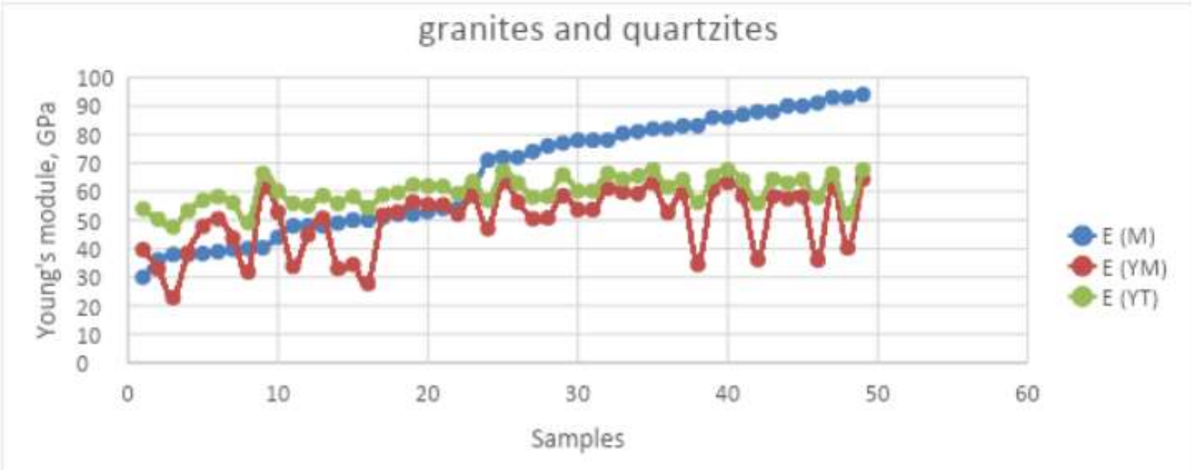


***Figure 9. Young's modulus variation of samples for granite and quartzite***

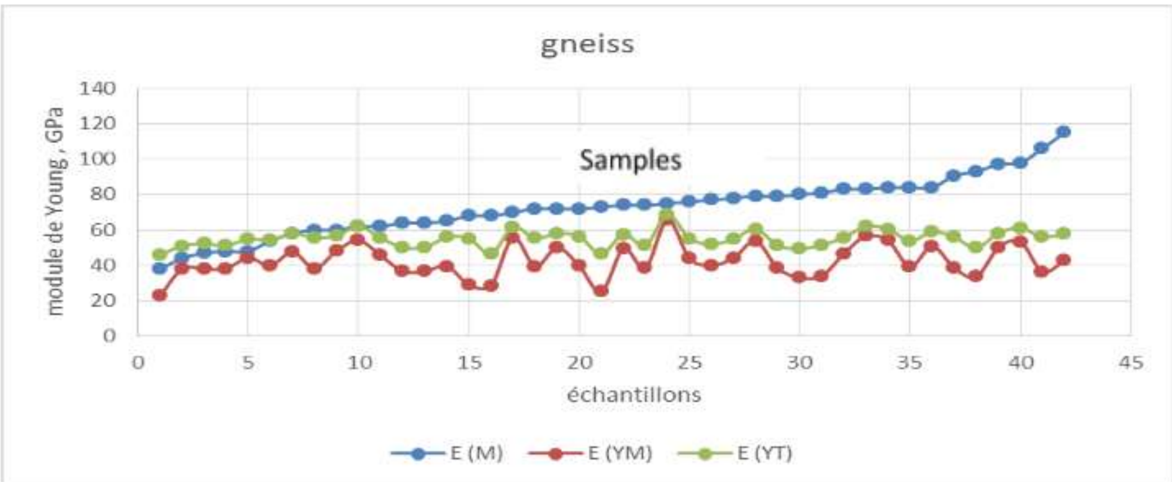


***Figure 10. Young's modulus variation of samples for gneisses***

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The differences between the measured and the determined values observed on the graphs present two behaviors distinct from the rocks: the rocks, whose Young's modulus is greater than 70 GPa and those, whose Young's modulus is lower than 70 GPa.

For rocks with a Young's modulus less than 70 GPa the deviations between the measured and determined values are small. The method of determining the modulus using the elemental values of the mineral modules (E (YT)) is more reliable for the gneiss group whereas the classical Mori-Tanaka method underestimates the Young's modulus values. For granites and quartzites the results obtained by the classical Mori-Tanaka method (E (YMT)) are closer to the measured values, whereas (E (YT)) overestimates these values.

For rocks with a Young's modulus greater than 70 GPa, both methods underestimate the true values and the gap increases with the resistance of the rocks.

## 4. CONCLUSION

The study presents the application of the porous media model (case of the Mori-Tanaka homogenization model) in order to determin the Young's modulus of granites, quartzites and gneisses taken from the quarries and outcrops of Togo. 91 rock samples including 49 granite and quartzite samples and 42 gneiss samples were studied

.

The results of studies of the correlations between the Young's modulus and the mineralogical, mechanical and physical characteristics of the rocks show that no characteristic allows the determination of the Young's modulus. The complexity of their mechanical behavior is related to the diversity of rocks from the point of view petrographic, textural, structural and physical. Nevertheless, some observations have been made:

- The decrease in% by volume of feldspar increases rock resistance to puncture, impact (LA) and wear (MDE) of rock aggregates,
- The Rock Drilling Test of LA Rock and MDE (the increase in rock resistance) increases Young's modulus.

Given the weak correlations between the Young's modulus and the characteristics of the rocks we propose the use of the classical and modified Mori-Tanaka methods applied to the three groups of rocks. For rocks with a Young's modulus of less than 70 GPa, low resistance rocks, the application of the modified Mori-Tanaka method is of interest in the determination of the Young's modulus of gneiss, whereas the classical Mori-Tanaka method gives satisfactory results for granites and quartzites. Neither method can be used to estimate the Young's modulus of granites, quartzites and gneisses, whose Young's modulus is high (> 70 GPa)

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